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VI Semester B.A./B.Sc. Degree Examination, September - 2021

MATHEMATICS

(CBCS Scheme Freshers &amp; Repeaters 2016-17 and onwards)

Paper : VIII

Time : 3 Hours

Maximum Marks : 70

*Instructions to Candidates:*

1. Answer ALL questions.
2. Non - programmable scientific calculators are allowed.

## PART - A

Answer any FIVE questions.

(5×2=10)

1.
  - a. Evaluate  $\lim_{Z \rightarrow 1+i} (Z^2 + 2Z)$ .
  - b. Find the equation of the circle with centre  $2-3i$  and radius 2 units.
  - c. Show that  $u = e^x \sin y + x^2 - y^2$  is harmonic function.
  - d. Show that  $f(z) = \sin x \cosh y + i \cos x \sinh y$  is analytic.
  - e. State Liouville's theorem.
  - f. Define cross ratio of four points.
  - g. Find the real root of the equation  $x^3 - x - 2 = 0$  over the interval  $(1.5, 2)$  upto two approximation by Bisection method.
  - h. State Runge - Kutta method of 4<sup>th</sup> order.

## PART - B

Answer FOUR full questions.

(4×10=40)

2.
  - a. Find the locus of the point  $Z$  satisfying  $\arg \left( \frac{z-1}{z+1} \right) = \frac{\pi}{3}$ .
  - b. State and prove necessary conditions for a function  $f(z) = u(x, y) + iv(x, y)$  to be analytic.

(OR)

[P.T.O.]



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3. a. If  $f(z) = u + iv$  is an analytic function, then prove that the curves  $u(x, y) = c_1$ ,  $v(x, y) = c_2$  form two orthogonal families.
- b. Show that  $f(z) = \cos z$  is analytic and hence show that  $f'(z) = -\sin z$ .
4. a. Find the analytic function  $f(z) = u + iv$  given that  $u - v = e^x (\cos y - \sin y)$ .
- b. Prove that  $u = y^3 - 3x^2 y$  is a harmonic function. Determine its harmonic conjugate.

(OR)

5. a. If  $f(z) = u + iv$  is analytic function, then show that  $\left[ \frac{\partial |f(z)|}{\partial x} \right]^2 + \left[ \frac{\partial |f(z)|}{\partial y} \right]^2 = |f'(z)|^2$ .
- b. Find the orthogonal trajectories of the family of curves  $e^{-x} \cos y + xy = c$ .
6. a. Evaluate  $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$  along
- the curve  $y = x^2$
  - the line joining  $(0, 1)$  and  $(2, 5)$ .
- b. State and prove the fundamental theorem of algebra.

(OR)

7. a. State and prove the Cauchy's integral theorem.
- b. Evaluate  $\int_c \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$ , where  $c$  is a circle  $|z| = 3$ .
8. a. Show that the bilinear transformation transforms circles into circles (or) straight lines.
- b. Discuss the transformation  $W = \sin z$ .

(OR)

9. a. Prove that the bilinear transformation preserves the cross ratio of four points.
- b. Find the bilinear transformation which map the points  $z = 1, i, -1$  into  $W = 2, i, -2$ .

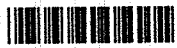
PART - C

Answer any TWO full questions.

(2×10=20)

10. a. Use Bisection method in five stages to find a real root of the equation  $x^3 - 2x - 5 = 0$ .
- b. Using Newton - Raphson method to find the real root of the equation  $x^3 + 5x - 11 = 0$  lies between 1 and 2, carry 3 iteration only.

(OR)



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11. a. Solve the equations by Gauss - Seidel method

$$10x + 2y + z = 9, x + 10y - z = -22, -2x + 3y + 10z = 22$$

b. Find the largest eigen value of the matrix  $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ .

12. a. Using Taylor's series method. Find  $y$  at  $x = 0.2$  given  $\frac{dy}{dx} = x - y^2$  and  $y(0) = 1$  upto the third degree.

b. Solve  $\frac{dy}{dx} = \frac{y-x}{y+x}$  with  $y(0) = 1$ . Find  $y$  for  $x = 0.1$  by Euler's method in five steps.

(OR)

13. a. Solve  $\frac{dy}{dx} = x + y$  with  $y(0) = 1$  for  $x = 0.1$  using Euler's modified method.

b. By using Runge - Kutta method of 4<sup>th</sup> order, Solve  $\frac{dy}{dx} = 3x + \frac{y}{2}$  with  $y(0) = 1$ , compute  $y(0.2)$  by taking  $h = 0.2$ .

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