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Reg. No. | | | | | | | | |

VI Semester B.A./B.Sc. Degree Examination, September - 2021
MATHEMATICS
(CBCS Scheme Freshers & Repeaters 2016-17 and onwards)
Paper : VIII

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer **ALL** questions.
2. Non - programmable scientific calculators are allowed.

PART - A

Answer any **FIVE** questions. **(5×2=10)**

1. a. Evaluate $\lim_{Z\to 1+i}(Z^2+2Z)$.

b. Find the equation of the circle with centre 2-3i and radius 2 units.

c. Show that $u = e^x \sin y + x^2 - y^2$ is harmonic function.

d. Show that $f(z) = \sin x \cosh y + i \cos x \sinh y$ is analytic.

e. State Liouville's theorem.

f. Define cross ratio of four points.

g. Find the real root of the equation $x^3 - x - 2 = 0$ over the interval (1.5,2) upto two approximation by Bisection method.

h. State Runge - Kutta method of 4th order.

PART - B

Answer **FOUR** full questions. **(4×10=40)**

2. a. Find the locus of the point Z satisfying $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{3}$.

b. State and prove necessary conditions for a function $f(z) = u(x,y) + iv(x,y)$ to be analytic.

(OR)

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3. a. If $f(z)=u+iv$ is an analytic function, then prove that the curves $u(x,y)=c_1$, $v(x,y)=c_2$ form two orthogonal families.

   b. Show that f(z) = cosz is analytic and hence show that $f'(z)=-Sinz$.

4. a. Find the analytic function $f(z)=u+iv$ given that $u-v=e^x(\cos y-\sin y)$.

   b. Prove that $u=y^3-3x^2y$ is a harmonic function. Determine its harmonic conjugate.

(OR)

5. a. If $f(z)=u+iv$ is analytic function, then show that $\left[\frac{\partial|f(z)|}{\partial x}\right]^2+\left[\frac{\partial|f(z)|}{\partial y}\right]^2=|f'(z)|^2$.

   b. Find the orthogonal trajectories of the family of curves $e^{-x}\cos y+xy=c$.

6. a. Evaluate $\int_{(0,1)}^{(2,5)}(3x+y)dx+(2y-x)dy$ along

   i. the curve $y=x^2+1$

   ii. the line joining (0,1) and (2,5).

   b. State and prove the fundamental theorem of algebra.

(OR)

7. a. State and prove the Cauchy's integral theorem.

   b. Evaluate $\int_c\frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)}dz$, where c is a circle $|z|=3$.

8. a. Show that the bilinear transformation transforms circles into circles (or) straight lines.

   b. Discuss the transformation W = Sinz.

(OR)

9. a. Prove that the bilinear transformation preserves the cross ratio of four points.

   b. Find the bilinear transformation which map the points $z=1,i,-1$ into W = 2, i, -2.

PART - C

Answer any **TWO** full questions. **(2×10=20)**

10. a. Use Bisection method in five stages to find a real root of the equation $x^3-2x-5=0$.

    b. Using Newton - Raphson method to find the real root of the equation $x^3+5x-11=0$ lies between 1 and 2, carry 3 iteration only.

(OR)

11. a. Solve the equations by Gauss - Seidel method

$10x+2y+z=9, x+10y-z=-22, -2x+3y+10z=22$

b. Find the largest eigen value of the matrix $A=\begin{bmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}$.

12. a. Using Taylor's series method. Find y at x = 0.2 given $\frac{dy}{dx}=x-y^2$ and y(0) = 1 upto the third degree.

b. Solve $\frac{dy}{dx}=\frac{y-x}{y+x}$ with y(0) = 1. Find y for x = 0.1 by Euler's method in five steps.

(OR)

13. a. Solve $\frac{dy}{dx}=x+y$ with y(0) = 1 for x = 0.1 using Euler's modified method.

b. By using Runge - Kutta method of 4th order, Solve $\frac{dy}{dx}=3x+\frac{y}{2}$ with y(0) =1, compute y(0.2) by taking h = 0.2.